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Mathematical models for piezoelectric energy harvesting

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Abstract: Energy problem has impact on the development. Fossil fuels are exhausted to fulfill our energy requirements. This contributes to environmental pollution. The global temperature is continuously rising. Global warming is a biggest problem today. Greenhouse gases are responsible for heating effect on the atmosphere; CO_2 is the major contributor. World's increasing demand of energy needs to be met with search for alternative renewable energy sources that do not lead to global warming. Based on energy sources, the form of electricity is the most practical energy used. Generating electricity is a major producer of carbon dioxide. Interest in providing electrical power for daily usage, from renewable sources has grown. Smart materials can give better substitute for traditional methods of energy generation. We can harvest energy using smart materials and simultaneously reduce CO_2 emissions. Thus smart materials are helpful in controlling carbon emission to atmosphere and have positive impact in controlling global warming. This paper reviews performances and developments of smart materials (like piezoelectric materials, piezo-composites and PZT) during energy harvesting from mechanical vibration. In this paper, energy harvesting with piezoelectric materials is highlighted as a solution for global warming. As mathematical model is a description of a system using mathematical concepts and language, so some mathematical models of vibration energy harvesting from piezoelectric materials are reviewed here.

Keywords: Global warming; Smart materials; Piezo-electric ceramic; PZT; Energy harvesting; Mechanical vibration; Mathematical model.

1. Introduction

The global temperature is continuously rising. It has been estimated that the global heat has increased approximately by 1°C since last 100 years. One solution to stop global warming is to control production of CO_2. This can be done by switching from hydrocarbons to renewable energy resources. The energy problem has impact on the developments. So Energy harvesting is the need of time.

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Energy harvesting means capturing energy from the energy sources from the surroundings, accumulating them. Extensive research has been conducted about the technology to harvest energy and use it as a self-power source for portable devices. Smart materials are employed to harvest energy.

Smart materials show extraordinary behavior when subjected to outer stimulus. Piezoelectric materials are considered as smart materials because of they generate electricity in proportion to mechanical stress applied. They develop electricity when mechanical stress is applied and vice versa. For piezoelectric materials the relation between applied mechanical forces with electrical charge generated is formulated as [(1–3)]

$$Q_i = e_{ijk} \sigma_{jk} \tag{1}$$

where Q_i is a component of the charge per unit / electric polarization, e_{ijk} are the components of piezoelectric coupling coefficient, and σ_{jk} are components of applied mechanical stress.

Although, energy harvesting have been used since years, but intensive research has been increased with concerns about global warming. With increase in global warming, piezoelectricity gained significant importance and extensive research and efforts are being made to develop technologies for extracting energy from the environment (See, 1, 3–9).

Silva combined piezoelectric and SMAs (shape memory alloys) for his proposed energy harvesting system (10). Umeda has successfully developed a model for PZT to transform energy due to mechanical impact into electrical power.

In this paper, piezoelectric energy harvesting is discussed in detail as a solution for global warming. This paper discussed energy harvesting technology from mechanical energy.

2. Energy harvesting using piezo-ceramics

Piezoelectric ceramics are great promise to energy harvesting. The work on many aspects of piezoelectric ceramics is carried out by researchers since a long time. Piezoelectric ceramics are being used for manufacturing sensors and actuators parts of smart structures. The work on energy harvesting aspects of such materials was started by stalwarts Mateu and Moll, Farinholt, Sohn, Koyama, Nakamura, Silva etc. in a big way. They developed various electric devices based on piezoelectric films. They analyzed various factors, like piezoelectric type, required energy, excitation magnitude, required voltage, and capacitor magnitude, to find a suitable choice of capacitor for storage and voltage intervals. Energy harvesting backpacks are developed by Farinholt et al. These backpacks convert mechanical forces (existing between the user and the backpack) into electrical energy. Sohn et al. adopted Finite Element Method to investigate the power harvesting under the influence of blood pressure by piezo films and analyzed it for circular and square configuration.

3. Mathematical models

The power output for piezoelectric films is generally predicted by Pin-force and Euler-Bernoulli methods. The main difference between these two models is in the consideration of the interaction between piezo-film and host structure. The predicted voltage outputs for Pin-force and Euler-Bernoulli are, respectively:

$$V = \frac{6g_{31}M}{bt_h(3-\varphi)}$$

and

$$V = -\frac{6g_{31}M\varphi(1+T)}{bt_a\left\{1+\varphi^2T^2+2\varphi(2+3T+2T^2)\right\}} \qquad (2)$$

where g_{31}, M, b, t_b, φ and T are the voltage constant of the piezo-film, moment, the width of the piezo-film, thickness, strain and the stress, respectively. Figure 1, shows the patterns for predicted voltage outputs by these two methods.

The general constitutive and gradient equations for piezoelectric ceramics are

$$\sigma_{ij} = c_{ijki}\gamma_{ki} - e_{kij}E_k \qquad (3)$$

$$D_j = e_{ijk}\gamma_{ij} + \varepsilon_{km}E_m \qquad (4)$$

$$\gamma_{ij} = \frac{1}{2}(u_i,_j + u_j,_i) \qquad (5)$$

$$E_i = -\phi,_i \qquad (6)$$

where σ_{ij}, γ_{ij}, D_i and E_i are the stress tensor, strain tensor, electrical displacement vector and electric field vector; u_i, ϕ, c_{ijkl}, e_{ijk} and ε_{ij} denotes the elastic displacement component, electric potential, elastic, piezoelectric and dielectric constants respectively. And comma denotes partial derivative w.r.to argument following it.

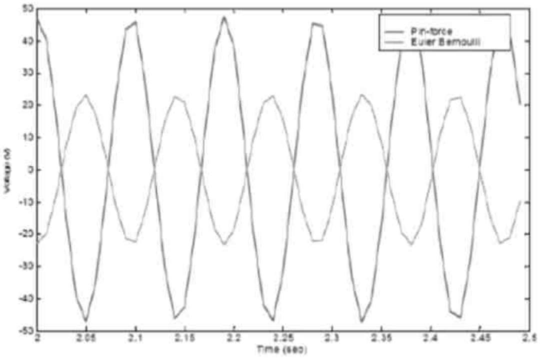

Fig. 1: The voltage calculated from Pin-force and Euler-Bernoulli models.

The general constitutive and gradient equations for poled transversely isotropic case get simplified and may be written in matrix form as

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{yz}\\ \sigma_{xz}\\ \sigma_{xy}\end{bmatrix}=\begin{bmatrix}c_{11} & c_{12} & c_{13} & 0 & 0 & 0\\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0\\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & c_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & c_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{(c_{11}-c_{12})}{2}\end{bmatrix}\begin{bmatrix}\gamma_{xx}\\ \gamma_{yy}\\ \gamma_{zz}\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{bmatrix}-\begin{bmatrix}0 & 0 & e_{31}\\ 0 & 0 & e_{31}\\ 0 & 0 & e_{33}\\ 0 & e_{15} & 0\\ e_{15} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}E_x\\ E_y\\ E_z\end{bmatrix} \tag{7}$$

$$\begin{bmatrix}D_x\\ D_y\\ D_z\end{bmatrix}=\begin{bmatrix}0 & 0 & 0 & 0 & e_{15} & 0\\ 0 & 0 & 0 & e_{15} & 0 & 0\\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\gamma_{xx}\\ \gamma_{yy}\\ \gamma_{zz}\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{bmatrix}+\begin{bmatrix}\varepsilon_{11} & 0 & 0\\ 0 & \varepsilon_{11} & 0\\ 0 & 0 & \varepsilon_{11}\end{bmatrix}\begin{bmatrix}E_x\\ E_y\\ E_z\end{bmatrix} \tag{8}$$

where z-axis is the poling direction.

These constitutive equations are further simplified as

$$\begin{aligned}&c_{44}\nabla^2 w+e_{15}\nabla^2\phi=0,\\ &e_{15}\nabla^2 w-\varepsilon_{11}\nabla^2\phi=0.\end{aligned} \tag{9}$$

Note that $\nabla^2\equiv\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ denotes Laplacian operator.

Using Fourier integral method the solution for (9) may be written as

$$w(x,y)=\frac{2}{\pi}\int_0^{\infty}\left\{\begin{array}{l}A_1(\alpha)e^{-\alpha y}\cos(\alpha x)\\ +\\ A_2(\alpha)\cosh(\alpha x)\sin(\alpha y)\end{array}\right\}d\alpha+a_x y \tag{10}$$

$$\phi(x,y)=\frac{2}{\pi}\int_0^{\infty}\left\{\begin{array}{l}B_1(\alpha)e^{-\alpha y}\cos(\alpha x)\\ +\\ B_2(\alpha)\cosh(\alpha x)\sin(\alpha y)\end{array}\right\}d\alpha-b_x y \tag{11}$$

Where $A_i(\alpha)$ and $B_i(\alpha)$ ($i=1,2$) are the functions to be determined and a_x, b_x are the arbitrary constants.

Conclusions

Use of rechargeable batteries is one of recent developments that motivates for power harvesting. These batteries are used to accumulate the generated energy during power harvesting. The rechargeable battery can be charged and then used to run any number of electronic devices for an extended period of time while being continuously charged by ambient motion. Innovations in power storage are required before power harvesting technology to be used at large scale.

In summary, the use of piezo-polymers for energy harvesting due to mechanical vibration is advantageous. The applications of piezo-polymer based energy-harvesters for wind; flower and backpack demonstrate its possible use in real life. Recently developed piezo-paper based on cellulose is another possibility for energy harvesting. Thus piezoelectric materials may substitute traditional energy sources to produce significant electric energy to power the necessary electronics. Since there is no burning process took place, so carbon dioxide is not produced during this process and hence smart materials are greener substitute. Furthermore practical applications of power harvesting systems and encouraging use of, renewable sources and smart materials, is beneficial for environment. It is helpful in reducing emission of CO_2 to the atmosphere and hence checks the global warming.

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