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Crack Arrest Model for Cracked Piezoelectromagnetic Strip under Electromagnetic Yielding

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Abstract: Crack arrest model for a piezoelectromagnetic strip under different magnetic and electrical yield conditions is proposed. The long narrow strip is cut along a transverse, internal, hairline straight crack. Infinite boundary of the strip is subjected to different sets of magnetic, mechanical and electrical load conditions. Consequently, the crack yields. A crack arrest is made possible by prescribing a magnetic, electric and mechanical load on rims of developed zones. Fourier integral transform is used to reduce the problem into dual integral equations. The solution of dual integral equations is then simplified numerically. Expressions are derived for energy release rate and load required to arrest developed zones.

Keywords: Crack Arrest; Piezoelectromagnetic; Transverse Crack; Fourier Integral Transform.

1. Introduction

The work on magnetoelectroelastic (MEE) fracture problem was started in the last century. The field is a natural extension of piezoelectric media since electricity and magnetism go in hand. Due to coupling effect of magneto, electro, and elastic fields, MEE materials become more popular than piezoelectric materials and serve as the excellent sensor, actuator and transducer. Wang and Shen [1] obtained energy release rate for a mode-III magnetoelectroelastic media based on the concept of energy-momentum tensor. Based on the extended Stroh formalism combined with complex variable technique, Green's function is obtained for an infinite two-dimensional anisotropic MEE media containing an elliptic cavity which degenerates into a slit crack, by Jinxi et al. [2]. Sih and Song [3] proposed a model which showed that crack growth in a magnetoelectroelastic material could be suppressed by increasing the magnitude of piezomagnetic constants in relation to these for piezoelectricity. They [4] further derived energy density function for cracked MEE medium and studied the additional magnetic-strictive effect which could influence crack initiation as applied field direction is altered. Wang and Mai [5], addressed the problem of a crack in a MEE medium possessing coupled piezoelectric, piezomagnetic, and magnetoelastic effects. Wang and Mai [6] further extended above problem to calculate a conservative integral based on governing equations for MEE media. Gao et al. [7] investigated the fracture mechanics for an elliptic cavity in a MEE solid under remotely applied uniform in-plane electromagnetic and/or antiplane mechanical loadings. Reducing cavity into a crack they considered two extreme cases for impermeable crack and permeable crack cases. Hu and Li [8] obtained singular stress, electric and magnetic fields in MEE strip containing a Griffith crack under longitudinal shear for a crack situated symmetrically and oriented in a direction parallel to the edges of the strip. Tian and Rajapakse [9] obtained the solution for single, multiple, and slowly growing impermeable cracks in a MEE solid using generalized edge dislocation theory. The solution for an elliptic cavity in an infinite two-dimensional MEE medium subjected to remotely apply uniform combined mechanical, electric, and magnetic loadings under permeable crack face boundary condition along the cavity of the surface had been obtained by Zhao et al. [10]. In this paper, we have proposed strip-yield-saturation-induction yield model for an unbounded cracked piezoelectromagnetic plate with electric and magnetic polarization in z-direction. Due to mechanical brittleness, it is assumed that developed mechanical yielding zone is the largest zone. In the problem, we consider the case when developed saturation zone is smaller than induction zone. Fourier transform technique is employed to obtain the solution and derived closed form expressions for developed zone sizes and energy release rate.

2. Formulation of the Problem

A long, narrow piezoelectromagnetic ceramic strip occupies the region $-h \le x \le h$ and $-\infty < y < \infty$ in *xoy* plane. The strip is thick enough in z-direction to allow the anti-plane shear state. The strip is poled along z-direction. Strip is cut along a thorough, finite, hairline, quasi-stationery, straight crack. The crack is symmetrically situated and transversely oriented with respect to the edges of the strip. The crack occupies the region $y = 0, -a \le x \le a$. The crack rims are stress and charge free. Also the edges of the strip are stress and charge free. The infinite boundary of the strip is prescribed uniform constant anti-plane shear stress and in-plane uniform constant unidirectional in-plane electric displacement and in-plane magnetic induction. Consequently, the crack rims open in self-similar fashion. Hence the crack rims open forming a strip-yield zone, a strip-saturation zone, and a strip-induction zone ahead of each tip of the crack. These strip zones are assumed to occupy the interval $a \le |x| \le b$, $a \le |x| \le c$ *and* $a \le |x| \le d$ on x-axis, respectively. To arrest the crack from further opening the developed saturation zone is subjected to normal, cohesive, unidirectional in-plane saturation limit electrical displacement, $D_y = D_s$, the rims of the developed slide zones are subjected to cohesive anti-plane yield point shear stress $\sigma_{yz} = \tau_s$ while developed induction zone rims are prescribed in-plane, normal cohesive saturation limit of magnetic induction, $B_y = B_s$. Entire configuration is schematically depicted in figure 1.

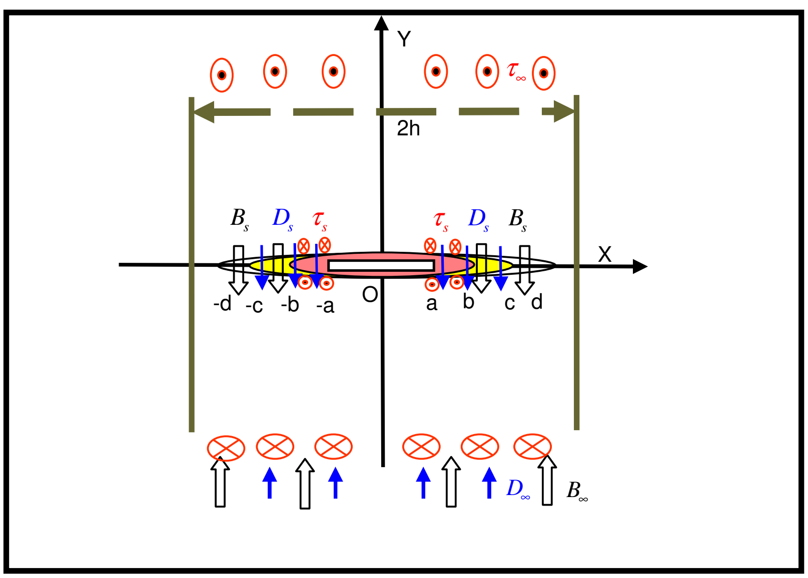

Figure 1: Schematic presentation of configuration of the problem.

Boundary conditions of the problem are:

(i) At infinite boundary of the strip i.e. for $|x| \le h$ and $y \to \infty$

$\sigma_{yz}(x,y) = \tau_\infty$ and $D_y(x,y) = D_\infty$ and $B_y(x,y) = B_\infty$

(ii) $\sigma_{yz}(x,0) = \tau_s H(x-a)$, $0 \le x < b$,

(iii) $D_y(x,0) = D_s H(x-a)$, $0 \le x < c$,

(iv) $B_y(x,0) = B_s H(x-a)$, $0 \le x < d$,

(v) $\phi_y(x,0^+) = \phi_y(x,0^-)$, $c \le x \le h$,

(vi) $u_z(x,0) = 0$, $b \le x \le h$,

(vii) $\psi_y(x,0^+) = \psi_y(x,0^-)$, $d \le x \le h$,

(vii) $D_x(h,y) = 0$,for all *y*,

(viii) $\sigma_{xz}(h,y) = 0$, for all *y*,

(ix) $B_y(h,y) = 0$, for all *y*.

Since the problem is symmetric in nature, only first quadrant is considered.

3. Fundamental Formulation

As is well-known the constitutive equations for out-of-plane displacement components $u_i(x,y,z)$, in-plane electric field component $E_i(x,y,z)$ and in-plane magnetic field component $E_i(x,y,z)$, $\{i = x,y,z\}$, may be written as

$$\sigma_{xz} = c_{44}u_z,_x + e_{15}\phi,_x + q_{15}\psi,_x ; \quad \sigma_{yz} = c_{44}u_z,_y + e_{15}\phi,_y + q_{15}\psi,_y \tag{1}$$

$$D_x = e_{15}u_z,_x - \varepsilon_{11}\phi,_x - d_{11}\psi,_x ; \quad D_y = e_{15}u_z,_y - \varepsilon_{11}\phi,_y - d_{11}\psi,_x \tag{2}$$

$$B_x = q_{15}u_z,_x - d_{11}\phi,_x - \mu_{11}\psi,_x ; \quad B_y = q_{15}u_z,_y - d_{11}\phi,_y - \mu_{11}\psi,_y \tag{3}$$

where σ_{iz}, D_i and B_i {*i=x,y*} denotes the shear stress, electrical displacement and magnetic induction components, respectively.

The governing equations for this case may be written as

$$\nabla^2 u_z = 0 \; ; \; \nabla^2\phi = 0 \text{ and } \nabla^2\psi = 0 . \tag{4}$$

These equations are solved using Fourier transform and taking inverse Fourier transform, solution may be written as

$$u_z(x,y) = \frac{2}{\pi}\int_0^\infty \left\{C_1(\alpha)e^{-\alpha y}\cos(\alpha x) + C_2(\alpha)\cosh(\alpha x)\sin(\alpha y)\right\}d\alpha + a_\infty y \tag{5}$$

$$\psi_1(x,y) = \frac{2}{\pi}\int_0^\infty \left\{B_1(\alpha)e^{-\alpha y}\cos(\alpha x) + B_2(\alpha)\cosh(\alpha x)\sin(\alpha y)\right\}d\alpha - b_\infty y \tag{6}$$

$$\psi_2(x,y) = \frac{2}{\pi}\int_0^\infty \left\{A_1(\alpha)e^{-\alpha y}\cos(\alpha x) + A_2(\alpha)\cosh(\alpha x)\sin(\alpha y)\right\}d\alpha - c_\infty y \tag{7}$$

where $C_i(\alpha)$, $B_i(\alpha)$ and $A_i(\alpha)$ ($i=1, 2$) are the arbitrary functions to be determined from the boundary conditions, of the problem.

$$\psi_1(x,y)=\phi(x,y)-\frac{(\mu_{11}e_{15}-d_{11}q_{15})}{\varepsilon_{11}\mu_{11}-d_{11}^2}\,u_z(x,y) \tag{8}$$

and,
$$\psi_2(x,y)=\psi(x,y)-\frac{(\varepsilon_{11}q_{15}-d_{11}e_{15})}{\varepsilon_{11}\mu_{11}-d_{11}^2}\,u_z(x,y) \tag{9}$$

Solution of the Problem

To obtain solution of the problem, equations (5) to (7) are simplified along with boundary conditions and after a long tedious calculation one arrives at the following explicit relations:

$$A_2(\alpha)=\frac{2}{\pi\alpha\sinh(\alpha h)}\int_0^\infty \frac{s\alpha}{s^2+\alpha^2}A_1(s)\sin(sh)ds \tag{10}$$

$$B_2(\alpha)=\frac{2}{\pi\alpha\sinh(\alpha h)}\int_0^\infty \frac{s\alpha}{s^2+\alpha^2}B_1(s)\sin(sh)ds \tag{11}$$

$$C_2(\alpha)=\frac{2}{\pi\alpha\sinh(\alpha h)}\int_0^\infty \frac{s\alpha}{s^2+\alpha^2}C_1(s)\sin(sh)ds \tag{12}$$

$$A_1(\alpha)=\frac{\pi b^2}{2}\int_0^1\sqrt{\xi}\,\phi_3(\xi)J_0(b\alpha\xi)d\xi \tag{13}$$

$$B_1(\alpha)=\frac{\pi c^2}{2}\int_0^1\sqrt{\xi}\,\phi_2(\xi)J_0(c\alpha\xi)d\xi \tag{14}$$

$$C_1(\alpha)=\frac{\pi d^2}{2}\int_0^1\sqrt{\xi}\,\phi_1(\xi)J_0(d\alpha\xi)d\xi \tag{15}$$

For $\phi_1(\xi)$, $\phi_2(\xi)$ and $\phi_3(\xi)$ following Fredholm integral equations of second kind are obtained:

$$\phi_1(\xi)+\int_0^1\phi_1(\eta)k_1(\xi,\eta)d\eta=\begin{cases} a_\infty\sqrt{\xi} & ,0<\xi<\dfrac{a}{d} \\ a_\infty\sqrt{\xi}+\tau_s-a_1+\dfrac{B_s}{a_2}\left(\dfrac{2}{\pi}\sin^{-1}\left(\dfrac{a\,\xi}{d}\right)-1\right) & ,\dfrac{a}{d}\le\xi<1 \end{cases} \tag{16}$$

$$\phi_2(\xi)+\int_0^1\phi_2(\eta)k_2(\xi,\eta)d\eta=\begin{cases} b_\infty\sqrt{\xi} & ,0<\xi<\dfrac{a}{c} \\ b_\infty\sqrt{\xi}+\tau_s-\dfrac{a_3}{a_2}\left(1-\dfrac{2}{\pi}\sin^{-1}\left(\dfrac{a\,\xi}{c}\right)\right) & ,\dfrac{a}{c}\le\xi<1 \end{cases} \tag{17}$$

$$\phi_3(\xi)+\int_0^1\phi_3(\eta)k_3(\xi,\eta)d\eta=\begin{cases} c_\infty\sqrt{\xi} & ,0<\xi<\dfrac{a}{b} \\ c_\infty\sqrt{\xi}+\tau_s-\dfrac{a_4}{a_2}\left(1-\dfrac{2}{\pi}\sin^{-1}\left(\dfrac{a\,\xi}{b}\right)\right) & ,\dfrac{a}{b}\le\xi<1 \end{cases} \tag{18}$$

Where,

$a_1=\dfrac{d_{11}(e_{15}(\varepsilon_{11}\mu_{11}-d_{11}^2)+\varepsilon_{11}q_{15}-d_{11}e_{15})}{\varepsilon_{11}\mu_{11}-d_{11}^2}$; $\qquad a_2=\varepsilon_{11}\mu_{11}-d_{11}^2$

$a_3=\mu_{11}D_s-d_{11}B_s$; $\qquad a_4=\varepsilon_{11}B_s-d_{11}D_s$

These equations are in turn solved numerically using quadrature methods. And hence problem is solved completely.

Energy release rate. Energy release rate, G, is calculated using

$$G = \frac{1}{2}\left[K_{III}^{\tau} K_{III}^{\gamma} - K_I^D K_I^E - K_I^B K_I^H \right]. \tag{19}$$

Where, K_{III}^{τ}, K_{III}^{γ}, K_I^D, K_I^E, K_I^B, K_I^H represents various required intensity factors and are given by

$$\begin{bmatrix} K_{III}^{\tau} \\ K_I^D \\ K_I^B \end{bmatrix} = \sqrt{\pi} \begin{bmatrix} \phi_1(1)\, \bar{c}_{44} \sqrt{d} \\ \phi_2(1)\, \varepsilon_{11} \sqrt{c} \\ \phi_3(1)\, \mu_{11} \sqrt{b} \end{bmatrix}; \qquad \begin{bmatrix} K_{III}^{\gamma} \\ K_I^E \\ K_I^H \end{bmatrix} = \begin{bmatrix} c_{44} & e_{15} & q_{15} \\ e_{15} & -\varepsilon_{11} & -d_{11} \\ q_{15} & -d_{11} & -\mu_{11} \end{bmatrix} \begin{bmatrix} K_{III}^{\tau} \\ K_I^D \\ K_I^B \end{bmatrix}$$

So, we get $G = \frac{\pi}{2}\left[a_5 (\phi_1(1))^2 + a_6 (\phi_2(1))^2 + a_7 (\phi_3(1))^2 \right]$ (20)

Where, $a_5 = \bar{c}_{44} d$; $a_6 = \frac{c(\varepsilon_{11})^2}{\bar{c}_{44}}\left[\frac{(q_{15})^2 + \mu_{11} c_{44}}{\varepsilon_{11}\mu_{11} - d_{11}^2} \right]$ and $a_7 = \frac{b(\mu_{11})^2}{\bar{c}_{44}}\left[\frac{(e_{15})^2 + \varepsilon_{11} c_{44}}{\varepsilon_{11}\mu_{11} - d_{11}^2} \right]$

Conclusions

The crack arrest model proposed above is more realistic model depiction of a situation for a cracked sensor/actuator/transducer with cracked narrow piezoelectromagnetic strip. The closed form expression is derived for energy release rate. Also the energy release rate behavior concludes that it is possible to slow down the crack growth. The results obtained can be applied to all ceramics sensors exhibiting the 6mm crystal symmetry along principal axis.

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