# Study of High Spin States in <sup>77-83</sup>Sr Isotopes

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**Abstract:** The isotopes of Sr have been studied by using the Projected shell Model (PSM). The Energy levels, transition energies have been calculated and compared with the available experimental data. The calculations reproduce the band head spins of positive parity yrast bands and indicate the multi-quasiparticle structure for these bands.

Keywords: Nuclear structure; Yrast spectra; Transition energy; Band diagram.

### 1. Introduction

The rare-earth nuclei have long served as the principal testing ground for the study of collective rotational behavior in nuclear physics. It is well known that on average the density of single particle energy level increases with increasing particle number, with spherical shell gaps at a neutron (N) or proton number = 2, 8, 20, 28, 50, 82 and 126. Shell gaps are also known to exist at larger deformation for different magic numbers. Clearly then, the level density for  $Z \approx 40$  is lower than for the rare-earth region for example, and, as a result, small change in the nucleon number, spin and excitation energy can have a relatively dramatic effect on the nuclear shape. Furthermore, it has become clear that nuclei around mass 80 are highly deformed [1-5] and can provide another fertile testing ground for rotational motion in nuclei. The proton-rich strontium isotopes have been known [4] to exhibit large prolate deformation at low excitation energy for some time with both proton and neutron numbers near the middle of the f-p-g shell, correlating to a relatively large number of valence particles, the ground states of these nuclei are primed to possess some of the strongest quadrupole deformation  $\beta_2$  in the mass 80 region [6]. The excited states in the light Sr isotopes were first established by Lister et al. [2]. Although <sup>77</sup>Sr is far from the stability, the ground-state spin and parity has been determined by beta decay[2] and laser spectroscopy [7] to be  $I^{\pi} = 5/2^+$ . Quadrupole moments deduced from lifetimes measured in <sup>79</sup>Sr [8] indicate very large deformation of  $\beta_2 \approx 0.4$  and are in agreement with theory [9]. In <sup>77</sup>Sr all negative parity levels and positive parity states above  $I = 13/2\hbar$  have been observed first time by C. J. Gross et al. [10]. The positive states extend up to  $I^{\pi} = (49/2^+)$ ; they are the highest spin states observed in any  $T_Z = \frac{1}{2}$  nucleus. The low-lying states in <sup>79</sup>Sr were identified by Lister et al. [11] by using a multiparticle  $\gamma$ -ray coincidence technique. The ground state band with band head  $3/2^{-}$  has been extended up to  $19/2^{-}$  and the positive parity band has been extended from  $5/2^{+}$  to  $15/2^{+}$ . The positive and negative-parity yrast sequences in <sup>79</sup>Sr have been established by Chishti et al.[12] up to probable spins 37/2<sup>+</sup> and 33/2<sup>-</sup>. Recently, Marginean et al. [13] has extended positive- and negativeparity band in <sup>79</sup>Sr from  $5/2^+$  to  $45/2^+$  and  $3/2^-$  ro  $45/2^-$ . Most recently Kaye et al. [14] has confirmed the existing level scheme of <sup>79</sup>Sr nucleus. High spin states in <sup>81</sup>Sr have been populated through the bombardment of a <sup>58</sup>Ni target with a <sup>36</sup>Si beam at 134MeV [15]. Isomeric states have been predicted [16] to exist in several bands in <sup>81</sup>Sr. A sudden cut-off in the  $\gamma$ -ray intensity at 5170.4keV for the  $\alpha$  = -1/2 sequence built on the  $[431]1/2^+$  Nilsson orbit is consistent with this state being isomeric. Arnell et al. [17] have first studied <sup>83</sup>Sr by <sup>80</sup>Kr ( $\alpha$ , n) <sup>83</sup>Sr reaction and Ekstrom [18] has extended the positive parity band from 7/2<sup>+</sup> up to 17/2<sup>+</sup>.

## 2 Projected Shell Model

The Projected Shell Model (PSM) has been designed and developed, so to speak, in order to meet the quality of measurements made possible by modern experimental techniques. The PSM is the natural extension of the SU(3) Shell Model for deformed system, where the Nilsson + BCS scheme is used for the basis selection and the projection of these deformed basis onto good angular momentum is done numerically. The deformed basis provides us an efficient way to describe the Shell Model basis. The PSM has been developed as a shell model truncation scheme which is implemented in a deformed single-particle basis [19]. Pairing correlations are included in this basis, which is constructed by the quasiparticle (qp) states obtained from a Nilsson + BCS calculation. The PSM proceeds as follows: first, the shell model truncation is carried out by considering the low-lying multi-qp configurations around the Fermi levels; then the angular-momentum-projection method is used to restore the rotational symmetry violated in the deformed basis. Finally, the two-body Hamiltonian is diagonalized in the projected basis. The truncation achieved in this way is very efficient.

The following set of multi-qp configurations is used for odd-neutron nuclei:

$$\left|\boldsymbol{\varphi}_{k}\right\rangle = \left\{a_{\nu}^{+}\left|\boldsymbol{0}\right\rangle, a_{\nu}^{+}a_{\pi1}^{+}a_{\pi2}^{+}\left|\boldsymbol{0}\right\rangle\right\},\tag{1}$$

where  $a^+$ 's are the qp creation operators and k labels each configuration. The states are written in the Nilsson + BCS representation, with v's ( $\pi$ 's) representing the neutron (proton) Nilsson quantum numbers, which run over low-lying orbitals, and  $|0\rangle$  the qp vacuum state. The 3-qp states are formed by one quasineutron plus a pair of quasiprotons. The inclusion of the 3-qp configurations is important for odd-mass nuclei for a description of the band-crossing phenomenon which is caused by a rotation alignment of a pair of quasineutrons. The Hamiltonian employed in the calculation is [19]

$$H = H_{o} - \frac{1}{2} \chi \sum_{\mu} Q_{\mu}^{+} Q_{\mu} - GP^{+}P - G_{Q} \sum_{\mu} P_{\mu}^{+} P_{\mu}$$
(2)

where  $H_0$  is the spherical single-particle Hamiltonian which contains a proper spin-orbit force. The second term in Eq. (2) is the quadrupole-quadrupole (*QQ*) interaction and  $\chi$  represents its strength, which is determined by the self-consistent relation between the input quadrupole deformation  $\varepsilon_2$  and the one resulting from the HFB procedure [19, 20]. The last two terms are the monopole and quadrupole pairing interactions, respectively.

The strengths of the monopole pairing interactions are given by

$$\mathbf{G}_{\mathrm{M}} = (G_1 \mp G_2 \frac{N-Z}{A}) \frac{1}{A},\tag{3}$$

and that for the quadrupole pairing interaction is related to the monopole pairing by

 $G_Q = \gamma \ G_M \tag{4}$ 

The Hamiltonian (2) is diagonalized in the shell model space spanned by  $\hat{P}_{MK}^{I} | \varphi_k \rangle$  where the  $\hat{P}_{MK}^{I}$  is the angular-momentum-projection operator and  $| \varphi_k \rangle$  the multi-qp states of eq. (1).

## **3** Results and Discussion

## 3.1 Yrast Spectra

In order to check the reliability of the PSM, we have done the calculations for the yrast states of <sup>77-83</sup>Sr isotopes. The comparison of the results of these calculations with the observed data has been presented in Fig.1[a-d]. From the Fig.1(a), we can see that in <sup>77</sup>Sr, the observed data[10] is quantitatively reproduced by our calculations up to the spin of  $23/2^+h$  and after that calculated values of the energy states are slightly above the experimentally observed values, upto the available observed spin  $49/2^{+}h$ . From Fig.1 (b), we can see that the observed data [13] in <sup>79</sup>Sr is exactly reproduced by our calculations up to the spin 25/2<sup>+</sup>ħ and for the higher states; the calculated states are slightly higher than the experimentally observed states. From fig. 1(c), we can see the agreement between the observed [21] and the calculated values of energy states in <sup>81</sup>Sr is very good up to the spin  $23/2^+\hbar$ , after this the difference between the two increases and the calculated energy values is very good up to spin  $19/2^{+}h$ . Then, after this spin, the values of calculated states are higher than the experimentally observed values up to the spin  $37/2^+h$  and beyond this spin, the difference between the two increases further. From Fig.1(d), we can see that the experimental energy states are qualitatively reproduced up to the spin 21/2 h, after it the calculated states are slightly higher than the observed values [22] up to the spin 37/2 h and thereafter the value of calculated states are further increased but the agreement between the two remain satisfactory even for the higher states.



Figure1(a-d): Comparison of the experimental and calculated yrast spectra in <sup>77-83</sup>Sr isotopes.

### **3.2** Transition energy

The plots of transition energy [E(I)-E(I-1)] versus spin (I) for <sup>77</sup>Sr, <sup>79</sup>Sr, <sup>81</sup>Sr and <sup>83</sup>Sr respectively are presented in Figures 2[a-d]. In case of <sup>77</sup>Sr, the calculated transition energy plot follows the same trend as followed by the experimental transition energy and are qualitatively reproduced at the lower spins. For <sup>79</sup>Sr, it can be noted that the variations shown by calculated transition energy values with spin agree qualitatively with the experimental values up to available spin of  $33/2^+\hbar$  and at the higher states it follows the behavior of the experimental one. For <sup>81</sup>Sr, it can be seen that there is a good agreement between calculated and experimental transition energies up to the spin value of  $21/2^+\hbar$ . Beyond this value of spin, the trend of the experimental transition energy is not reproduced well. Now, in case of <sup>83</sup>Sr, there is an overall satisfactory agreement between the experimental and theoretical transition energies.



Figure 2[a-d]: Comparison of the Experimental and calculated transiton energies in <sup>77-83</sup>Sr isotopes.

### 3.3 Band Diagrams

In this paper, we have calculated another nuclear structure property of Sr isotopes and that is the band diagram. The band diagrams have been presented in the Fig.3 [a-d]. From Fig.3(a), we can see that in <sup>77</sup>Sr, the 1-qp state with configuration  $1\nu g9/2$ ,[5/2] k = 5/2 acts as the yrast band up to spin  $35/2^{+}\hbar$ . Beyond the spin  $35/2^{+}\hbar$ , there is mixing of two bands having configuration  $1\nu g9/2$ ,[-7/2] k = -7/2 and  $1\nu g9/2$ ,[5/2] +  $2\pi g 9/2$ ,[-3/2, 5/2] k = 7/2 and they collectively contribute towards the formation of the yrast band of <sup>77</sup>Sr. In case of <sup>79</sup>Sr, we can see from fig.3(b), that the 1-qp state with configuration  $1\nu g9/2$ , [5/2] with band head k= 5/2 is acting as the yrast band up to  $31/2^{+}\hbar$  and after this spin the 3-qp band of configuration  $1\nu g 9/2$ , [5/2] +  $2\pi g 9/2$ ,[-3/2, 5/2] +  $2\pi g 9/2$ ,[-3/2, 5/2] with k = 7/2 crosses band with k = 5/2 and then, this band acts as the yrast band in higher spin states. From the band diagram of <sup>81</sup>Sr [Figure 3(c)], it is observed that the energy states from  $7/2^{+}\hbar$  to  $11/2^{+}\hbar$  of the yrast band arise from one neutron  $1\nu g9/2$ , [-7/2] k=-7/2 quasiparticle (qp) state. Between the spins  $15/2^{+}\hbar$  to  $21/2^{+}\hbar$ , there is

mixing of two 1-qp bands having configuration 1vg9/2, [-7/2] k=-7/2 and 1vg9/2, [5/2] k=5/2. From spins  $23/2^{+}\hbar$  to  $43/2^{+}\hbar$ , there is mixing of two 3-qp bands and then, beyond this, the spectra is arising from the 3-qp band having configuration 1vg9/2,  $[-7/2] + 2\pi g9/2$ , [-3/2, 1/2] k=-9/2. In case of <sup>83</sup>Sr, again one neutron 1vg9/2, [-7/2] k=-7/2 band represents yrast states for spins  $7/2^{+}\hbar$  to  $17/2^{+}\hbar$ . After the spin value of  $17/2^{+}\hbar$ , there is mixing of different bands up to the spin  $41/2^{+}\hbar$  and then afterwards the 3-qp band having configuration 1vg9/2,  $[1/2]+2\pi g9/2$ , [-3/2,5/2] k=3/2 becomes the yrast band. Thus, from the graphs presented in Fig. 3[a-d], we get that the yrast states are not composed of the single-qp states but there is the contribution of 3-qp states and mixing of the both toward the formation of the yrast states at higher spin.



Figure 3[a-d]: Band Diagrams in<sup>77-83</sup>Sr isotopes

## 4 Conclusions

From the results of the calculations the conclusions drawn are that the PSM calculations of yrast bands for <sup>77</sup>Sr and <sup>79</sup>Sr show a very good agreement with the corresponding experimentally observed bands for all the available values of spins. In <sup>81</sup>Sr and <sup>83</sup>Sr, the calculated yrast bands are in good agreement with the experimentally observed bands at low spins. But at higher spins, the calculated values are slightly higher than the experimental ones. The PSM calculations of transition energies for all the isotopes show good agreement, qualitatively and quantitatively, with the experimentally observed data. It is also observed from the calculations that the yrast band in these nuclei does not arise from 1-qp state only but also from 3-qp states.

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