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Microscopic study of deformation systematics in neutron-deficient (N < 50) strontium isotopes

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Abstract: In the present piece of work, the trend of the onset of deformation in neutron-deficient $^{80-84}$Sr isotopes in Z = 38, N<50 region and the factors which are contributing towards it, in a suitable microscopic calculational framework known as Variation after projection, are studied. The nuclear structure quantities like Yrast spectra, *B*(*E*2) transition probabilities, Quadrupole deformation parameter (β_2) and sub-shell occupation numbers for even-even neutron-deficient $^{80-84}$Sr isotopes are calculated in the HB-framework for the trial wave functions resulting from the pairing-plus-quadrupole-quadrupole-plus-hexadecapole-hexadecapole (PQH) interaction.

Key words: Yrast spectra, *B*(*E*2) transition probabilities; quadrupole deformation parameter (β_2).

1. Introduction

The nuclei in the mass region A = 60 - 90 have been the subject of extensive experimental and theoretical treatment in the last many years. These nuclei exhibit a variety of nuclear phenomena like co-existence of shapes, large ground state deformation, band crossing, rapid variation of structure with changing nuclear number, etc. The study of low-and high-spin phenomena in the neutron-deficient mass-80 nuclei has also attracted considerable interest in recent years. This has been motivated by the increasing power of experimental facilities and improved theoretical descriptions, as well as by the astrophysical requirement in understanding the structure of these unstable nuclei. In comparison to the rare-earth region where the change in nuclear structure properties is quite smooth with respect to particle number, the structure of neutron-deficient mass-80 nuclei shows considerable variations when going from one nucleus to another. This is mainly due to the fact that the available shell model configuration space in the mass-80 region is much smaller than in the rare-earth region. The low single particle level density implies that a drastic change near the Fermi surfaces can occur among neighbouring nuclei. Another fact is that in these medium mass neutron-deficient nuclei, neutrons and protons occupy the same single particle orbits. Some extensive measurements [1-18] of the transition quadrupole-moments, extracted from the level lifetimes and excitation energy of 2^+ state, have been carried out for neutron-deficient Sr nuclei.

These measurements have revealed large prolate deformation in the neutron-deficient Sr nuclei and have also reported large variations in nuclear structure of these isotopes with respect to particle number and angular momentum. It has been shown that alignment of proton and neutron pairs of higher angular momenta can change the nuclear shape from prolate to triaxial and to oblate.

The light neutron-deficient even-even Sr isotopes with A = 80 - 84 are known as some of the most deformed in the mass A = 80 region [11]. The neutron-deficient Sr isotopes have been predicted [14, 15] to have large deformations because of the gap in the single particle energy levels at Z=38. Deformations of $\beta \approx 0.38$ have been determined for Sr isotopes using life-time measurements [11, 12, 16] and isotope shifts [13, 17, 18]. Despite a band crossing, ^{80}Sr has been shown [16] to display a very gradual alignment and the B(E2) values indicate a constant deformation throughout the reported spin range. Further, it has also been found that $^{82-84}$Sr are located at the boundary between the spherical and deformed strontium isotopes [19, 20]. The transitional nuclei $^{82-84}$Sr exhibit complex behaviour that indicates an interplay between several degrees of freedom such as collective rotational and vibrational excitations, single particle excitations and core deformation. The neutron-deficient strontium isotopes appear to evolve from spherical to highly deformed as neutrons are removed from valence shell.

The low-spin level scheme for ^{80}Sr was investigated via ^{80}Y decay by Doring and co-workers [21]. The high spin states in ^{80}Sr have been recently investigated by Winchell et al. [22] with the reaction ^{58}Ni (^{28}Si, α 2p).

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Previous work on ^{80}Sr by Davie et al. [16] established the high-spin decay scheme of the ground state band to a spin of 26$\hbar$.

Life-time measurements of high spin states in ^{82}Sr indicate a possible loss of collectivity in the ground state band of that nucleus [23]. In early 1970's, a work on ^{82}Sr was done with heavy-ion [24] and (p,t) reactions [25]. The fact that the 4^+ state was not seen in the (p,t) reactions implied that this might be a soft nucleus with structural changes occurring even at low spin. In early 1980's, different groups [26-28] studied ^{82}Sr with a variety of heavy-ion reactions and implied that ^{82}Sr is a vibrational nucleus.

High spin states in ^{84}Sr have been experimentally investigated by G. Garcia Bermudez et al. [29] through the ^{52}Cr(^{36}S,2p2n) reaction. Previous work on the study of high spin states in ^{84}Sr by Dewald et al. [30] used the conventional in-beam gamma ray spectroscopy with (HI, xn) reactions and found a close-lying 8^+ triplet state above the yrast 6^+ state. Low-spin states of this nucleus were also studied by Rester et al. [31] using the (α, α') experiments and by Ball et al. [25] using (p,t) reactions.

An interesting feature of the observed energy spectra (see Table 3(b).1) is the variation of E_2^+ excitation energy from ^{80}Sr to ^{84}Sr. For example, whereas the value of E_2^+ excitation energy in case of ^{80}Sr is 0.38MeV, it increases to 0.79 MeV for ^{84}Sr. The ratio of excitation energies of the first 4^+ to 2^+ states is also known to be a guide to the size and type of deformation. The value of E_4^+/E_2^+ ratio goes on decreasing from 2.58 for ^{80}Sr to 2.22 for ^{84}Sr. It has been pointed out by Zhang et al. [32] that the ratio E_4^+/E_2^+ is an important parameter for determining the shape of a nucleus. For a rigid rotator, its value should be 3.33 whereas its value for a spherical nucleus should be around 2. The values in between these two limits indicate that the nucleus is quasi-deformed and has vibrational character. Thus, from the observed spectra in neutron-deficient $^{80-84}$Sr isotopes, it appears that ^{80}Sr is most deformed and there is a decrease in the degree of deformation as one move from ^{80}Sr towards ^{84}Sr, which is least deformed among these nuclei. The observed trend of deformation in neutron-deficient Sr isotopes is also confirmed by the systematics of B (E2; $0_1^+ \rightarrow 2_1^+$) values given in Table 3(b).7. Whereas for ^{80}Sr, the B (E2; $0_1^+ \rightarrow 2_1^+$) value is of the order of $0.95e^2b_n^2$, it decreases to 0.28 $e^2b_n^2$ for ^{84}Sr. It seems that for the isotopes $^{80-84}$Sr , there is an empirical relationship between B (E2; $0_1^+ \rightarrow 2_1^+$) and the excitation energy of the 2_1^+ state (ΔE). As ΔE increases, B (E2; $0_1^+ \rightarrow 2_1^+$) decreases in its absolute value. This type of inverse correlation between B (E2; $0_1^+ \rightarrow 2_1^+$) values and ΔE is in fact consistent with the Grodzins rule [33]. Grodzins has indeed developed a quantitative formula relating the two quantities. Raman et al. [34] recently updated Grodzins work based on all current data.

From the systematic analysis of the experimental results, it is quite clear that the isotopes $^{80-84}$Sr have the following features:-

(i) The deformation is the maximum in ^{80}Sr in this region.
(ii) The neutron-deficient light Sr nuclei with A<82 have prolate shape in the ground state.
(iii) The transitional nuclei $^{82-84}$Sr exhibit co-existence of spherical and prolate shapes.
(iv) Transition of spherical shape to prolate shape takes place over ^{84}Sr and ^{82}Sr.

Apart from the experimental study, a large number of theoretical calculations have also been performed to study various properties of neutron-deficient Sr isotopes. The early mean-field approaches were devoted to a general study of the structure in the mass-80 region. Besides the work using the Woods-Saxon approach [35], there were studies using the Nilsson model [36] and the Skyrme +Hartree-Fock + BCS theory [15]. More recently, microscopic calculations were performed using the Excited VAMPIR approach [37, 38]. However, this approach is numerically quite involved and is usually employed to study some quantities for selected nuclei. The large scale spherical shell model diagonalization calculations [39] have been recently successful in describing the pf-shell nuclei, but the configuration space required for studying the well-deformed mass-80 nuclei is far beyond what the modern computers can handle.

A recent theoretical analysis has predicted a non-collective yrast state at spin 16$\hbar$ [40]. Several superdeformed bands populated above 30$\hbar$ have also been reported in ^{80}Sr [41].The phenomenological shell model calculations have been carried out for $^{80-86}$Sr by Ogawa [42] and Kitching et al. [43]. Their calculations were successful for ^{86}Sr but the description of the isotopes $^{80-84}$Sr was less successful, even for the low-spin states. An IBM calculation has been performed by Dewald et al. [30] for ^{84}Sr to explain the low-spin states and observed shape transition. The structure of the collective bands in ^{84}Sr is also investigated within the framework of the deformed configuration–mixing shell model based on Hartree-Fock states[20]. Recently, the projected shell model (PSM) [44] has been applied for the study of the yrast band structure of the neutron-deficient Sr nuclei in the mass-80 region. This model has been successful in explaining different nuclear properties of $^{76-80}$Sr nuclei in the mass-80 region. Further, the collective-cum-microscopic model [45, 46] was also applied by a number of workers to carry out the calculations over the whole isotopic chart.

From the overview of the theoretical work, it has been observed that various theoretical approaches, which have been applied on the study of even-even neutron-deficient Sr nuclei in the mass-80 region, so far, are found to explain successfully only a few isotopes in the $^{80-84}$Sr isotopic mass chain and hence, the clarity regarding the overall systematics of Sr isotopes is not there. Further, though, theoretical interest has continued but still a great deal of theoretical work is required to understand the details of the systematics of low-lying states and trends of deformation in these nuclei. Thus for a complete analysis, there is a need of a more elaborate microscopic nuclear theory.

In view of this, in the present piece of work, we have studied the trend of the onset of deformation in neutron-deficient $^{80-84}$Sr isotopes in Z = 38, N<50 region and the factors which are contributing towards it, in a suitable microscopic calculational framework. As it was shown by Khosa and Sharma [47], two body effective interactions have a dominantly quadrupole-quadrupole character and the deformation producing tendency of neutron-proton (np) and like particle interactions depends upon the degeneracy of the underlying single particle valence space. One of the natural choices for the two-body residual interaction would, therefore, be pairing plus quadrupole-quadrupole (PQ) interaction. We have, however, found it important to include a correction energy term to the PQ interaction in the form of hexadecapole-hexadecapole interaction, which hereafter will be denoted as PQH interaction. We have, thus, calculated various nuclear structure quantities like Yrast spectra, $B(E2)$ transition probabilities, Quadrupole deformation parameter (β_2) and sub-shell occupation numbers for even-even neutron-deficient $^{80-84}$Sr isotopes in the HB-framework for the trial wave functions resulting from the pairing-plus-quadrupole-quadrupole-plus-hexadecapole-hexadecapole (PQH) interaction, operating in a valence space spanned by $3s_{1/2}$, $2p_{1/2}$, $2p_{3/2}$, $2d_{3/2}$, $2d_{5/2}$, $1f_{5/2}$, $1g_{7/2}$, $1g_{9/2}$ and $1h_{11/2}$ orbits for protons and neutrons. The doubly closed nucleus ^{56}Ni has been taken as inert core.

2. Material and Methods

The details of calculational framework are same as given Ref. [47].

2.1 The one- and two-body parts of the Hamiltonian

In these calculations, We have employed the valence space spanned by $3s_{1/2}$, $2p_{1/2}$, $2p_{3/2}$, $2d_{3/2}$, $2d_{5/2}$, $1f_{5/2}$, $1g_{7/2}$, $1g_{9/2}$ and $1h_{11/2}$ orbits for protons and neutrons under the assumption of N = Z = 28 sub-shell closure. The single particle energies (S.P.E.'s) that we have taken are (in MeV) : $(3s_{1/2})$ = 9.90, $(2p_{1/2})$ = 1.08 ,($2p_{3/2}$) = 0.0 , $(2d_{3/2})$ =11.40, $(2d_{5/2})$ = 8.90 , $(1f_{5/2})$ = 0.78 , $(1g_{7/2})$ =11.90 , $(1g_{9/2})$ =3.50 and $(1h_{11/2})$ =12.90. The energy values of single particle orbits for $2p$-$1f$-$1g$ levels are the same as employed for ^{56}Ni core plus one nucleon. The energies of higher single particle valence orbits are the same as used by Vergados and Kuo [48] relative to $1g_{9/2}$ valence orbit.The two- body effective interaction that has been employed here is of PQH type. The pairing–plus-quadrupole-quadrupole (PQ) interaction is of the type given in reference [49]. The strength for the like particle neutron-neutron (n-n), proton-proton (p-p) and neutron-proton (n-p) components of the quadrupole-quadrupole (q-q) interaction were taken as:

$$\chi_{nn} \; (= \chi_{pp}) = -\,0.0100 \text{ MeV } b^{-4}$$

$$\chi_{np} = -\,0.0188 \text{ MeV } b^{-4}$$

Here b (= $\sqrt{\hbar/m\omega}$) is the oscillator parameter. The strength for the pairing interaction was fixed through the approximate relation G = (18-21)/A.

We have carried out the calculations by incorporating hexadecapole–hexadecapole interaction term in the PQ two-body interaction. The relative magnitudes of the parameters of the hexadecapole-hexadecapole parts of the two- body interaction were calculated from a relation suggested by Bohr and Mottelson [50]. According to them, the approximate magnitude of these coupling constants for isospin T = 0 is given by

$$\chi_\lambda = \frac{4\pi}{2\lambda+1}\frac{m\omega_0^2}{A<r^{2\lambda-2}>} \text{ for } \lambda = 1,2,3,4 \qquad \ldots\ldots \; (1)$$

and the parameters for the T=1 case are approximately half the magnitude of their T = 0 counterparts. This relation was used to calculate the values of χ_{pp4} relative to χ_{pp} by generating the wave-function for Strontium isotopes and then calculating the values of $<r^{2\lambda-2}>$ for λ = 2 and 4.

The values for hexadecapole-hexadecapole part of the two body interaction turn out to be

$$\chi_{pp4} \; (= \chi_{nn4}) = -0.00032 \text{ MeV } b^{-8}, \text{ and}$$

$$\chi_{pn4} = -0.00064 \text{ MeV } b^{-8}$$

Here, b (= $\sqrt{\hbar / m\omega}$) is the oscillator parameter. The strength for the pairing interaction was fixed through the approximate relation G = (18-21)/A.

3. Results and discussion

3.1 Deformation trends in $^{80-84}$sr:

We first discuss here the systematics of E_2^+ excitation energy in $^{80-84}$Sr (See Table 1). It has been observed that the energy of 2_1^+ (ΔE) increases from 0.38MeV for ^{80}Sr to 0.79MeV for ^{84}Sr. This systematic increase in the value of ΔE with 'A' gives an indication that there is a decrease in the degree of deformation as we move from ^{80}Sr to ^{84}Sr. Phenomenologically, it is well known from Grodzins rule [33] that a nucleus having a smaller energy gap ΔE should have a larger quadrupole moment for 2^+ state. Since quadrupole moment of second excited state (Q_2^+) of a nucleus is related to its intrinsic quadrupole moment, one should, therefore, expect that a smaller energy gap ΔE should manifest itself in terms of a larger quadrupole moment and vice-versa. In other words, the observed systematics of E_2^+ with '*A*' should produce a corresponding inverse systematics of intrinsic quadrupole moments of Sr nuclei with increasing 'A'. Based on the above logic, the calculated values of intrinsic quadrupole moments should exhibit a decreasing trend as we move from ^{80}Sr to ^{84}Sr. This trend is exhibited by the calculated intrinsic quadrupole moments. In Table1, the results of HFB calculations obtained with PQH interaction are presented. It may be noted that the value of intrinsic quadrupole moment goes on decreasing from 78.27 b^2 in ^{80}Sr to 39.44 b^2 in ^{84}Sr, where b = $\sqrt{\hbar}$ /mω is the oscillator parameter. This decrease in the value of intrinsic quadrupole moment from ^{80}Sr to ^{84}Sr indicates that there is a decrease in the degree of deformation from ^{80}Sr to ^{84}Sr. This fact is also confirmed by the decrease in the value of the ratio E_4^+/E_2^+ (given in Table 1). The value of this ratio decreases from 2.58 for ^{80}Sr to 2.22 for ^{84}Sr. Thus, we can say that the results on the intrinsic quadrupole moments are consistent with E_2^+ systematics.

Table 1. The experimental values of excitation energy of the E_2^+ state**(**ΔE) in MeV, intrinsic quadrupole moments of proton ($<Q_0^2>_\pi$) , neutron($<Q_0^2>_\upsilon$) and the HB states $<Q_0^2>_{HB}$ obtained with PQH interaction and the experimental values of E_4^+/E_2^+ ratio for $^{80\text{-}84}$Sr isotopes. The quadrupole moments have been computed in units of b^2, where $b=\sqrt{\hbar / m\omega}$ is the oscillator parameter.

Sr-Nuclei (A)	(Expt.) E_2^+	PQH Interaction			(Expt.) E_4^+/E_2^+
		$<Q_0^2>_\pi$	$<Q_0^2>_\upsilon$	$<Q_0^2>_{HB}$	
80	0.38*	37.24	41.03	78.27	2.58*
82	0.57**	27.37	32.05	49.42	2.31**
84	0.79***	20.91	18.53	39.44	2.22***

* Data taken from Ref. [22], **Data taken from Ref. [23], ***Data taken from Ref. [29]

Attention is next focused on the factors that are responsible for making the Sr-isotopes to exhibit such features. In this regard, it is important to discuss and highlight some of the well-accepted factors responsible for bringing in sizeable collectivity in the nuclei in general. It is well known that if the down slopping components of a high-*j* valence orbit starts filling up, it has the effect of bringing in sharp increase in collectivity. Besides this, it is also known that a closed shell or an empty sub-shell makes zero contribution to the intrinsic quadrupole moment. Therefore, if a sub-shell gets polarized and still has occupation probability greater than the mid sub- shell occupation, then it will again have the effect of introducing some degree of deformation in the nucleus. In addition, the role of n-p interaction in the SOP (spin-orbit partner) orbits in the context of the general development of collective features was also suggested by Federman and coworkers [51,52] and by Casten et al. [53]. Their calculation provided evidence suggesting the neutron-proton interaction between the valence nucleons in the SOP orbits- the orbits $(g_{9/2})_\pi$ and $(g_{7/2})_\upsilon$ in the Zr and Mo region- may be instrumental *vis-à-vis*, the observed onset of deformation in the neutron-deficient Sr-isotopes with N<50. The subscript π stands for proton and υ stands for neutron. In the light of above effects, it is now tried to find out the causes responsible for the observed systematics of $^{80\text{-}84}$Sr-isotopes.

In Tables 2 and 3, the results of occupation probabilities of various proton and neutron sub-shells for the ground state calculated from HB wave-functions with PQH interaction for $^{80\text{-}84}$Sr-isotopes are presented. It may be noted from these Tables that for ^{80}Sr, $(p_{3/2})_\pi$ orbit is maximally polarized as its occupation is 2.21 units. As one moves away from ^{80}Sr towards ^{84}Sr, the $(p_{3/2})_\pi$ orbit occupation goes on increasing. For example, the occupation of $(p_{3/2})_\pi$ orbit for ^{84}Sr is 3.18 units. Besides this, the occupation of $(g_{9/2})_\pi$ orbit goes on decreasing in its value from 2.68 units for ^{80}Sr to 2.00 units for ^{84}Sr. These changes in the occupation of $(p_{3/2})_\pi$ and $(g_{9/2})_\pi$ orbits are responsible for the decrease of $<Q_0^2>_\pi$ as one moves from ^{80}Sr towards the higher mass side. Further, from Table 3, it is also noted that $(p_{1/2})_\upsilon$, $(p_{3/2})_\upsilon$, $(f_{5/2})_\upsilon$ and $(g_{9/2})_\upsilon$ occupations increase in their values as one moves from ^{80}Sr to ^{84}Sr. This systematic change in the occupation of these orbits is responsible for the decrease of $<Q_0^2>_\upsilon$ as one moves from ^{80}Sr to ^{84}Sr. From Tables 2 and 3, it is also clear that for ^{84}Sr, the neutron sub-shells $p_{1/2}$, $p_{3/2}$, and $f_{5/2}$ are nearly full and $(g_{9/2})_\upsilon$ orbit is more than half-filled. Besides this, the $(p_{3/2})_\pi$ orbit is more than two-third full for ^{84}Sr.Because of the closure of most of the neutron sub-shells in the valence space and the $(p_{3/2})_\pi$ sub-shell being more than two-third full, the quadrupole moment of ^{84}Sr is less than that of the other Sr isotopes and therefore, it has nearly spherical structure. The overall observed deformation systematics in $^{80\text{-}84}$Sr could be understood in terms of the systematic changes in the occupation probabilities of the various valence orbits as explained above.

Table 2. The sub - shell occupation numbers (protons) in the nuclei $^{80\text{-}84}$Sr obtained with PQH interaction

Sr Nuclei (A)	Sub-shell occupation number								
	$3s_{1/2}$	$2p_{1/2}$	$2p_{3/2}$	$2d_{3/2}$	$2d_{5/2}$	$1f_{5/2}$	$1g_{7/2}$	$1g_{9/2}$	$1h_{11/2}$
80	0.18	0.54	2.21	0.14	0.86	3.21	0.11	2.68	0.03
82	0.10	0.59	2.84	0.06	0.45	3.31	0.04	2.47	0.09
84	0.06	0.72	3.18	0.03	0.29	3.57	0.03	2.00	0.07

Table 3 The sub - shell occupation numbers (neutrons) in the nuclei $^{80\text{-}84}$Sr obtained with PQH interaction

Sr Nuclei (A)	Sub-shell occupation number								
	$3s_{1/2}$	$2p_{1/2}$	$2p_{3/2}$	$2d_{3/2}$	$2d_{5/2}$	$1f_{5/2}$	$1g_{7/2}$	$1g_{9/2}$	$1h_{11/2}$
80	0.40	0.73	3.01	0.59	0.94	3.72	0.37	4.20	0.00
82	0.11	1.66	3.85	0.08	0.55	5.44	0.06	4.24	0.02
84	0.06	1.97	3.97	0.03	0.4	5.94	0.03	5.55	0.07

Now the attention is focused on the observed trend of deformation in $^{80\text{-}84}$Sr-isotopes in the light of np-interaction operating between SOP(spin-orbit partner) orbits. Note that there is an increase in the occupation of $(p_{3/2})_\pi$ orbit from its value of 2.21 units for ^{80}Sr to 3.18 units for ^{84}Sr and also there is an increase in the occupation of $(p_{1/2})_\upsilon$ orbit from 0.73 units for ^{80}Sr to 1.97 units for ^{84}Sr. Thus, there is an increased opportunity for np-interaction between the SOP-orbits, the $(p_{3/2})_\pi$ and $(p_{1/2})_\upsilon$ orbits for ^{80}Sr, to operate effectively. Thereafter, this opportunity of np-interaction goes on decreasing as one moves away from ^{80}Sr to ^{84}Sr. Hence, the np-interaction between SOP orbits is maximum for ^{80}Sr and this np-interaction goes on decreasing as one moves away from ^{80}Sr towards ^{84}Sr. Therefore, ^{80}Sr is maximally deformed and this deformation goes on decreasing from ^{80}Sr to ^{84}Sr. All these factors are, therefore, responsible for the decrease in the degree of deformation in Sr-isotopes lying in the mass-80 region with Z=38 and N<50.

3.2. *Yrast spectra*

In order to test the reliability and efficiency of HB calculations performed with PQH model of two-body interaction, it is important to obtain a satisfactory agreement for the Yrast spectra. So, in this regard, a calculation for the energy spectra of $^{80\text{-}84}$Sr was carried out by employing the phenomenological PQ and PQH models of two-body interaction. Starting from the Hamiltonian ($H - \beta Q_0^2$),the even spin and even parity energy states were obtained for each Strontium isotope. In Figures 1 (a)-1(c), the calculated values of energy (E_J) corresponding to each angular momentum state (J^π) are presented in the form of Yrast spectra for ^{80}Sr to ^{84}Sr. The Yrast spectra obtained with PQ interaction is denoted by Th.1 and the Yrast spectra obtained with PQH model of two-body interaction is denoted by Th.2. The calculated Yrast spectra corresponding to Th.1 and

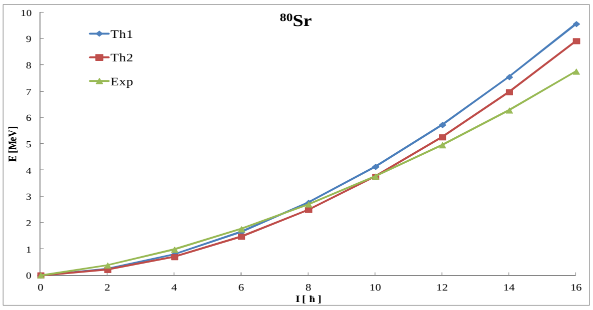

Fig. 1(a)

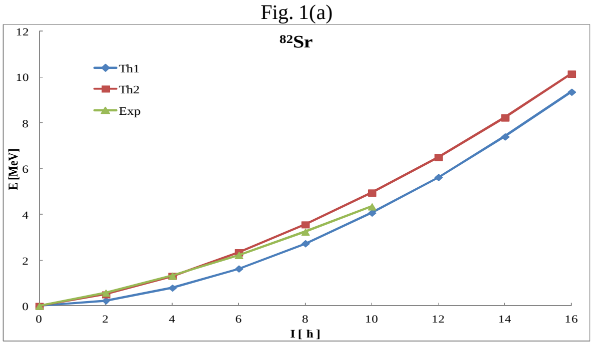

Fig. 1(b)

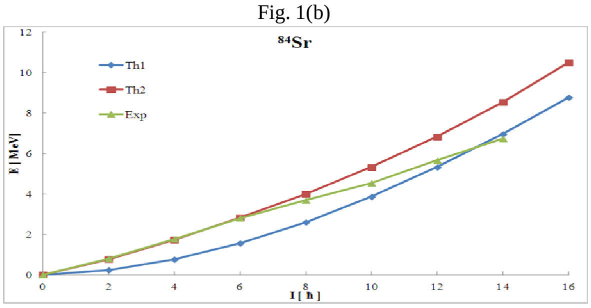

Fig. 1 (c)

Figure 1. Comparison of the experimental*(Expt.) and calculated(Th.) values of the energy (E_J) of the low-lying yrast states (in MeV) obtained with PQ(Th.1) and PQH(Th.2) interactions (a) for ^{80}Sr isotope, *Data taken from Ref.[22], (b) for ^{82}Sr isotope, *Data taken from Ref.[23], (c) for ^{84}Sr isotope, *Data taken from Ref.[29].

Th.2 are also compared with the experimental Yrast spectra for $^{80-84}$Sr isotopes. From Figures 1(a-c), it is clear that all the observed Yrast states for $^{80-84}$Sr are very well reproduced by the calculated Yrast spectra corresponding Th.2 and have better agreement than the spectra corresponding to Th.1. Thus, it turns out from the calculations that the overall agreement of the observed Yrast spectra for $^{80-84}$Sr with the calculated Yrast spectra is obtained provided the calculations of Yrast spectra are made with PQH interaction as there is a lot of improvement in the calculated Yrast spectra when one goes from the spectra corresponding to Th.1 to the spectra corresponding to Th.2.

Hence, it is concluded from the study of Yrast spectra in $^{80-84}$Sr isotopes that PQH model of two-body interaction is an improvement over the PQ model of two-body interaction in case of neutron-deficient Sr isotopes.

3.3 Transition probabilities

The reliability and goodness of the HB wave function is also examined by calculating the $B\ (E2;\ 0_1^+ \rightarrow 2_1^+)$ values. Sometimes back, Bhatt et al. [54] have developed a formula for the calculation of $B(E2;\ 0_1^+ \rightarrow 2_1^+)$ transition probabilities from the values of intrinsic quadrupole moments of protons and neutrons. It has been justified by them that the $B\ (E2;\ 0_1^+ \rightarrow 2_1^+)$ in units of $e^2\ b_n^2$ (where b_n stands for barn,1barn = 10^{-28}m^2) are given by:

$$B\ (E2;0_1^+ \rightarrow 2_1^+) = (1.02\times10^{-5})A^{2/3}\, c^2_{model}\, [e_\pi \langle Q^2_0\rangle_\pi + e_\nu \langle Q^2_0\rangle_\nu]^2 \quad .....2$$

where $\langle Q^2_0\rangle_\pi$ ($\langle Q^2_0\rangle_\nu$) are the intrinsic quadrupole moments of valence protons (neutrons) and e_π and e_ν are the effective charges of the protons and neutrons, respectively.

Effective charges of the protons and neutrons are 1.3 and 0.3 respectively for $^{80-84}$Sr.

The value of c_{model} = (1.11±0.27) for 28 ≤ Z ≤ 50 [55]. I have used this formula with c_{model} = 1.20 for the calculation of the $B(E2)$ values for the mass chain of $^{80-84}$Sr isotopes.

In Table 4, a comparison of the calculated $B(E2)$ values obtained with PQH interaction and the experimental values [57] for the $0_1^+ \rightarrow 2_1^+$ transitions in case of $^{80-84}$Sr isotopes is presented. The $B(E2)$ values corresponding to PQH interaction have been calculated by using intrinsic quadrupole moments given in Table1. It may be noted that the calculated $B(E2)$ values are in good agreement with the experimental values for the $0_1^+ \rightarrow 2_1^+$ transitions in case of $^{80-84}$Sr when the effective charges for protons and neutrons are used as 1.3 and 0.3 respectively. The use of effective charges [56] is generally invoked in nuclear structure calculations to represent the contribution made by the core towards the electromagnetic properties, due to its getting polarized as the nucleons are put in the valence space. The valence particles through two-body effective interactions can interact with the core and cause excitations. The degree of polarization of the core is, therefore, expected to increase with increase in the number of valence particles. Since the calculations for the $B(E2)$ values depend on the intrinsic quadrupole moments, so $B(E2)$ values should follow the same trend as that followed by the intrinsic quadrupole moments . This feature of the neutron-deficient Strontium isotopes has been reproduced by the present calculations.

Table 4. Comparison of the experimental and calculated B(E2; $0_1^+ \rightarrow 2_1^+$) values Quadrupole deformation parameter (β_2) in $^{80-84}$Sr isotopes obtained with PQH interaction. The B(E2) values are in units of $e^2 b_n^2$(where b_n stands for barn, 1barn = 10^{-28}m^2).

Sr-Nuclei (A)	B(E2;0_1^+□2_1^+) Th.	B(E2;0_1^+□2_1^+) (Expt)*	β_2 Th.	β_2 (Expt)*
80	0.97	0.95(36)	0.43	0.40(8)
82	0.54	0.51(20)	0.30	0.29(6)
84	0.29	0.28(44)	0.22	0.21(16)

*Data taken from Ref. [57]

3.4 Quadrupole deformations (β_2)

In Table 4, the calculated values for deformation parameter (β_2) have also been presented. The deformation parameter β_2 is related to $B(E2)\uparrow$ by the formula suggested by Raman et al. [55],

$$\beta_2 = (4\pi / 3\ Z\ R_0^2)\ [B(E2)\uparrow/e^2]^{1/2} \qquad .... (3)$$

where R_0 is usually taken to be 1.2 $A^{1/3}$ fm and $B(E2)\uparrow$ is in units of $e^2\, b_n^2$.

From the systematics of the calculated β_2 values, it is noted that the agreement between the set of values obtained with PQH interaction and the observed values is satisfactory.

4. Conclusions

Based on the results of present calculations, the following conclusions can be drawn:-

(i) The most deformed character of ^{80}Sr is due to the occupation of $g_{9/2}$ proton orbit and $p_{1/2}$, $p_{3/2}$, $f_{5/2}$ and $g_{9/2}$ neutron orbits. Besides this, $p_{3/2}$ proton orbit is maximally polarized.

(ii) The observed decrease in the degree of deformation as one move away from ^{80}Sr towards ^{84}Sr is seen to be dependent on the decrease in the occupation of $(g_{9/2})_\pi$ orbit and the increase in the occupation of $(p_{3/2})_\pi$ orbit. These changes in the occupation of

$(p_{3/2})_\pi$ and $(g_{9/2})_\pi$ orbits are responsible for the decrease of intrinsic quadrupole moment $\langle Q_0^2 \rangle_\pi$ as one moves from ^{80}Sr to ^{84}Sr. Besides this, the increase in the occupation of $(p_{1/2})_\upsilon$, $(p_{3/2})_\upsilon$, $(f_{5/2})_\upsilon$ and $(g_{9/2})_\upsilon$ orbits is responsible for the decrease of $\langle Q_0^2 \rangle_\upsilon$ as one moves from ^{80}Sr to ^{84}Sr.

(iii) The observed decrease in the degree of deformation in $^{80-84}$Sr isotopes is also dependent on the decreased opportunity for the np-interaction between the SOP orbits-the $(p_{3/2})_\pi$ and $(p_{1/2})_\upsilon$ orbits, to operate effectively.

(iv) The Yrast spectra obtained with the inclusion of hexadecapole interaction i.e., PQH interaction show a good agreement with the observed Yrast spectra as compared to the Yrast spectra obtained with PQ model of two-body interaction.

(v) The values of B(E2) transition probabilities and quadrupole deformation parameter (β_2) calculated with PQH interaction for $^{80-84}$Sr are found to be in satisfactory agreement with the experiments.

Hence, it is concluded that the values of hexadecapole interaction parameters employed here, for $^{80-84}$Sr isotopes, are the appropriate ones in this mass region as, with them, the HB wave-function yields various nuclear structure quantities which are in a satisfactory agreement with the experiments.

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